

Digital Wave Prints

In “Catching Waves: Making Wave Prints to Introduce Wave Properties” Laurie Kaup and her colleagues at the University of Hawaii at Manoa described a method for capturing the profile of a wave by dipping a sheet of paper in a water tank with a standing wave.

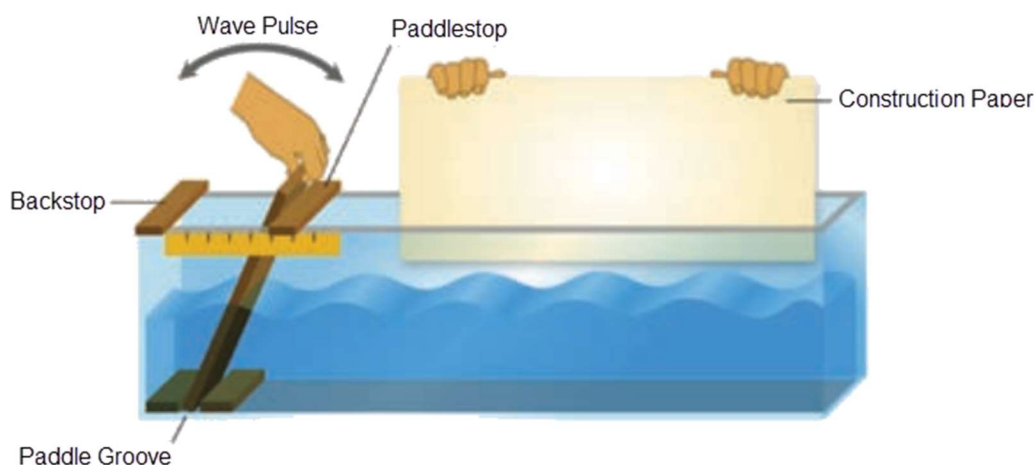


Figure 1. Creating a Wave Print – Kaup, et al., 2013 (reproduced with permission of the author)

A paddle at one end of the tank was used to generate a standing wave. The outline of the damp area of paper dipped in the water was then used to trace a profile of the wave. Kaup used the term “wave print” to describe the profile captured in this manner. The wave print was used to measure the amplitude and wavelength of the wave.

This method was piloted in thirty middle school classrooms in Hawaii. The results were published in a 2013 issue of *Science Scope*, a peer-reviewed practitioner journal for science teachers published by the National Science Teachers Association (Kaup, Philippoff, Seraphin & Lurie, 2013). The activity was subsequently republished with permission of the authors in the science section of the *Educational CAD Model Library* (Kaup, Philippoff, Seraphin & Lurie, 2023). The *CAD Library* is a peer-reviewed repository of educational objects (Bull, et al., 2023).

Capturing a Digital Profile of a Wave

The widespread prevalence of smart phones today offers the opportunity to capture digital wave prints using images of waves captured with the high-resolution cameras found in most phones. The wave print activity was developed for an acrylic wave tank that was 1.4 m long, 30 cm high, and 12 cm wide. A custom aquarium builder provided a quotation of \$1,500 to build a wave tank with the same dimensions. A local glass and mirror firm provided the acrylic pieces with these dimensions for \$200. Local fabrication of the wave tank can yield significant savings when staff time permits, but there are also a number of custom aquarium builders who can fabricate a wave tank if funds are available.

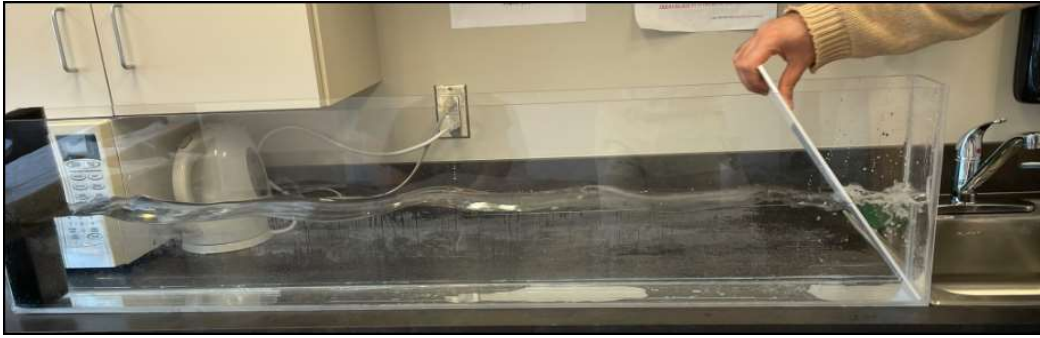


Figure 2. A Wave Tank Fabricated with Quarter-Inch Acrylic

We used Weldon #4 acrylic cement to bond the pieces of quarter-inch thick acrylic together. After allowing the cement to dry overnight, we then sealed the seams with a silicone gel to ensure that the wave tank was water tight. Following the directions provided in the wave print article, we placed a paddle groove on the bottom of the tank and fabricated five- and ten-centimeter paddle stops placed at the top of the tank.

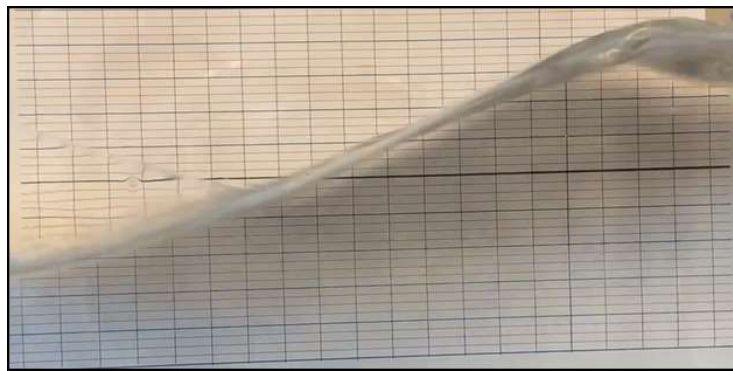


Figure 3. Using a Grid to Measure Wave Amplitude and Wavelength

A tripod was used to position the smart phone in front of the wave tank. A video recording was used to capture the motion of the wave. The video recording was used to scan the recording frame-by-frame to identify a frame in which the maximum and minimum height of the wave could be seen. The minimum and maximum height of the wave were then measured using a grid on a transparent sheet of plastic that had been taped to the front of the tank. The wavelength was also recorded by using the grid to measure the distance between two peaks.

Analog-to-Digital Conversion

Use of a camera to capture the profile of a wave can address concepts found in the Next Generation Science Standards (NGSS). The NGSS standard MS-PS4-3 *Waves and their Applications in Technologies for Information Transfer* states that students should be able to integrate scientific information “to support the claim that digitized signals are a more reliable way to encode and transmit information than analog signals.”

In contrast to earlier analog cameras that captured images on film, the cameras in smart phones are digital. Sampling rate and resolution are two of the relevant factors that affect the quality of the image in a video. Many phones can capture video in 4K resolution (3840 x 2160 pixels) at 60 frames per second. Some phones can capture video at 8K resolution (7680 x 4320 pixels) at 120 frames per second. At lower

frame rates, the motion of the wave may be blurred. Initial tests indicated that 60 frames per second was sufficient to freeze the wave and capture usable images of the wave profile.

Sources of Error in Digital Measurements

The resolution of an image determines the granularity with which measurements can be made. At 4K resolution, we were able to make usable measurements with horizontal grid lines spaced at quarter-centimeter intervals. The vertical grid lines were spaced at one-centimeter intervals. This resolution and spacing made it possible to measure the amplitude of waves with one-eighth centimeter accuracy by interpolating between grid lines (i.e., by estimating whether the wave peak was closer to a grid line or whether it fell in the middle of two grid lines).

The distance at which the cell phone camera was placed from the wave tank also affected the accuracy with which measurements could be made. The cell phone camera was placed far enough from the wave tank to make it possible to clearly see two full waves (i.e., three wave peaks). The focal length of the camera is a factor that affects its field of view and the positioning of the camera. The focal length of the camera used was 24 mm. The positioning of a camera with a different focal length to obtain the same field of view of the wave tank could differ. The camera must be placed parallel to the wave tank to avoid distortion known as parallax.

The process of converting an analog signal into a digital one is known as analog-to-digital conversion. For example, the microphone in a laptop computer generates an electrical voltage as the back-and-forth movement of the diaphragm of the microphone is generated by the pressure produced by soundwaves. The varying voltage is an analog signal. An analog-to-digital converter in the computer measures voltage level at specified intervals and converts this measure into a number stored in a table in the computer.

The sample rate for most digitized music is 44,100 samples per second (44.1 KHz). The sample resolution is determined by the number of voltage levels sampled, which in turn is determined by the number of bits recorded. For example, an 8-bit sample can store 256 different combinations, while a 16-bit sample can store 65,536 levels. Increased sample rate and bit-depth increases the accuracy of the digitized signal.

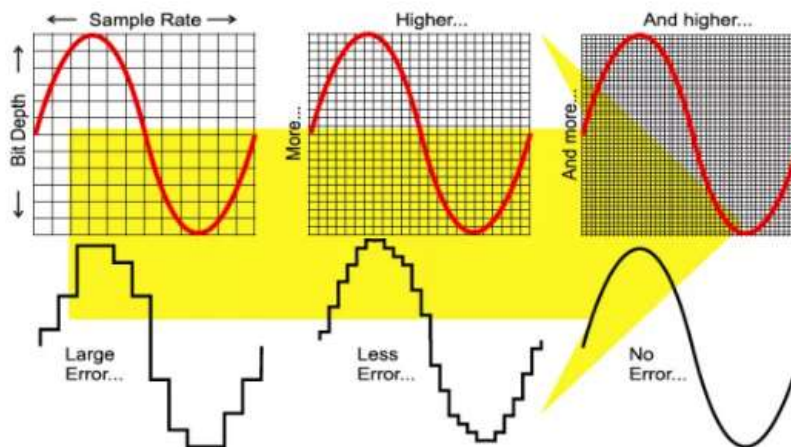


Figure 4. Relationship between Sampling Rate, Bit Depth and Quantization Error

In practice, music digitized at 44 KHz with a resolution of 24 bits per sample is sufficient to satisfy anyone except for the most discerning audiophile.

In the case of a wave in the wave tank, the sample rate and resolution required are lower than the sample rate and resolution required for an audio signal by orders of magnitude. However, the principles involved in the digitization process are the same. In contrast to a sound wave, a water wave is visible. It provides a useful entry point for exploring the digitization process.

Sources of Error in Analog Measurements

Analog measurements also have inherent limitations. In the case of wave prints, a team of three students dips the construction paper into the water to create a wave profile:

“One person creates waves, another watches the side of the tank for a standing wave, and a third person dips the paper. The ‘dipper’ has to be told when to dip, because it is hard to see the standing wave when looking straight down into the tank.” (Kaup, 2013, p 7.)

The timing required to estimate when the wave is at peak amplitude can introduce error. For one condition of the reported results (for a 5 cm paddle deflection at 100 beats per minute), a peak amplitude of 2.0 cm was recorded for the first trial and a peak amplitude of 2.6 cm was recorded for the second trial. The recorded variation between trials was 30% in this instance. After the paper is dipped into the water, it must be removed before the results are obscured by the next wave. Thus, the measurements are affected by the skill of the operator and the inherent limitations of human reflexes.

The video of wave motion can be scanned to find the frame at which the wave peak is at its maximum, potentially removing the physical skill of the observer as a factor that introduces error. However, as noted above, the digitization process can introduce other types of error. Comparison of analog and digital methods provides insight into factors that are common to many processes involving measurement and address the NGSS standards related to digital recording and transmission of information.

Comparing Analog and Digital Measurements

In order to form conclusions regarding analog versus digital measurements, a class could first create an analog wave print by dipping a sheet of paper in water. Students could then measure amplitude and wave length of a wave by creating a digital wave print. Questions to consider are whether there are discernable differences in the accuracy of measurements obtained with analog versus digital wave prints.

If there are discernable differences, students can then form conclusions about whether the differences are meaningful. The Kaup *Science Scope* article (2013) reported the following results:

- as frequency increases, wavelength decreases;
- at the same paddle-stop setting, wave height increases as frequency increases; and
- as paddle-stop setting distance increases with constant frequency, wave height increases.

Students can explore whether the same results are obtained with both analog and digital wave prints and discuss advantages and limitations inherent in both methods. Any time that a signal is digitized or a scientific measurement is obtained digitally, there are opportunities for errors in the digitization process.

Parallels with Waves in Different Mediums

A number of different mechanical models have been developed to demonstrate and explain wave motion. The *Shive Wave Machine* was developed at Bell Laboratories (Shive, 1961) to demonstrate concepts such as frequency, period, wavelength, and amplitude in a mechanical wave system. The wave machine consists of 72 eighteen-inch long steel rods mounted on bearings around a central axis.

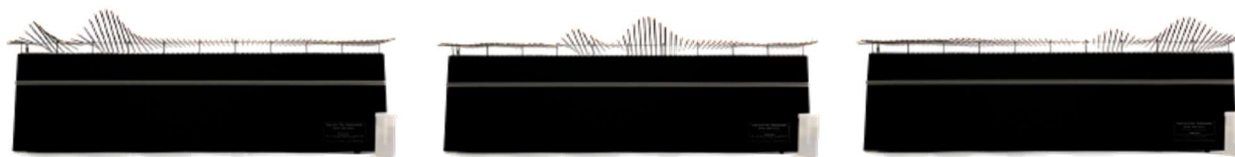


Figure 5. Transmission of a Wave across the Length of a Mechanical Wave Machine

Movement of a rod at one end of the wave machine is transmitted down the length of the machine. For example, when the wave reaches the end of the wave machine, it is reflected and returns in the opposite direction. When pulses are transmitted at a constant rate, the amplitude of the wave is greater for some frequencies than for others. The frequency at which the amplitude of the wave is greatest is known as the *resonant frequency*.

A guide, *Similarities in Wave Behavior* (Shives, 1961), explains that many properties of wave motion hold true across waves in mechanical, electrical, acoustic, and optical systems. Consequently, it might be expected that waves in a wave tank would also have a frequency at which the amplitude is greatest, and that the amplitudes of waves greater or less than this optimal frequency would be lower.

This can be verified experimentally through use of either analog or digital measurements. While some variations between the two methods might occur due to different sources of error such as the ones described above, it might be expected the general pattern would hold for both methods. As Shives notes, any freely vibrating object in nature – whether it is a violin string, a vibrating column of air in a wind instrument, a pendulum, or a swing on a playground – will have a resonant frequency at which the amplitude of the back-and-forth movement is the greatest.

A larger vibrating mass typically has a lower resonant frequency than a smaller one. A cello string has a lower rate of vibration than a violin string and a tuba creates a lower tone than a coronet. Similarly, a larger speaker cone responds more readily to lower frequencies while a smaller cone responds more readily to higher frequencies. Students at Buford Middle School in Charlottesville constructed speakers of different sizes and then determined the resonant frequency for each type of speaker.



Figure 6. Analyzing the Frequency Response of Speakers of Different Sizes

The students used a sound level meter to measure the sound pressure level of the speaker cone across a range of frequencies. They found that the resonant frequency of the larger speaker peaked in the lower frequency range (below 500 Hz).

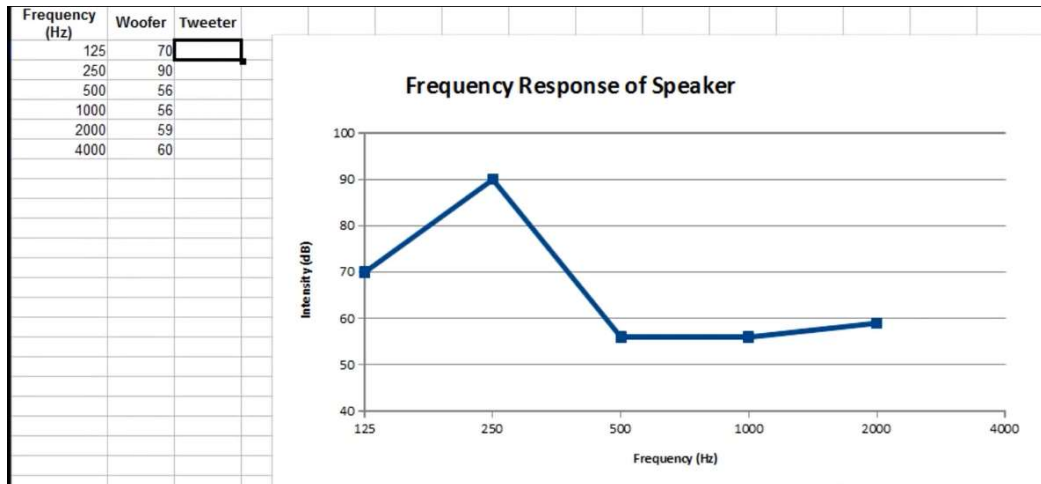


Figure 7. Frequency Response Curve of a Larger Speaker (Woofers)

In contrast, they found that the peak amplitude for a smaller speaker took place at a higher frequency.

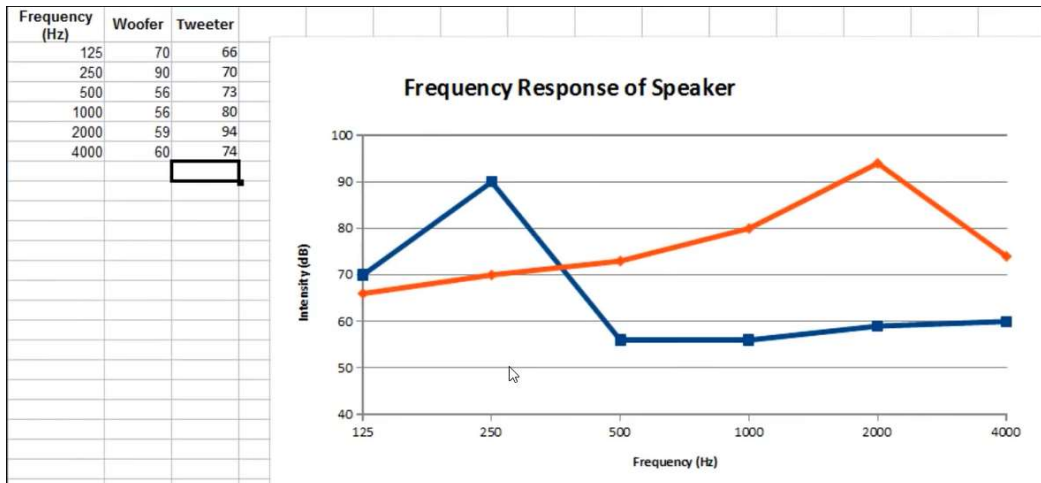


Figure 7. Frequency Response Curve of a Smaller Speaker (Tweeters – Shown in Red)

The difference in frequency response related to the mass of the speaker (as well as other factors) is why a high-fidelity speaker system will often incorporate two or more speakers of different sizes. The manufacturer usually provides a frequency response curve for the system that can be used to evaluate the fidelity of the system.

In another context, an audiogram measures the response of the human auditory system to tones of different frequencies. Often as people age, there is a decrease in hearing acuity for higher frequencies.

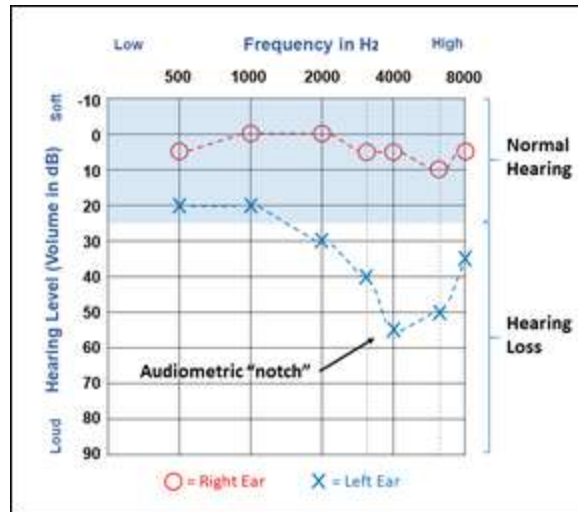


Figure 8. Audiogram Depicting Normal Hearing (Right Ear) and Hearing Loss (Left Ear)

Pilot Digital Wave Profile Results

The pilot results for the peak-to-peak amplitude, from the highest peak to the lowest trough, of waves in the wave tank exhibited a similar pattern. The amplitude of waves increased from a rate of 40 beats per minute until a maximum amplitude of about 9 centimeters was reached at 90 beats per minute. The amplitude then gradually decreased from 90 beats per minute through 180 beats per minute.

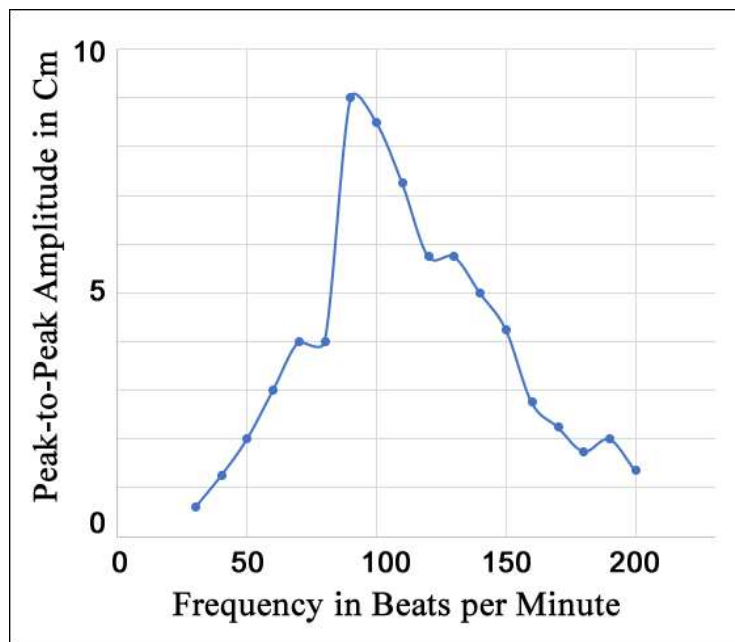


Figure 9. Wave Amplitude for 40 to 180 Beats per Minute

At some frequencies, reflected waves interfere with new waves generated by the movement of the wave paddle. The interference between the two sets of waves reduces the overall amplitude of the waves. At other frequencies, reflected waves coincide and align with one another, causing the overall amplitude of the waves to increase. The point of maximum reinforcement is described as the resonant frequency.

The waves recorded were asymmetrical in height, with greater displacement above the midline formed by the level of the water at rest than below the midline. When the depth of the water is less than half the length of the waves measured horizontally from peak to peak, reflections from the bottom also affect waves on the surface. This can be seen in a dramatic way during the transition from shallow coastal waters to deeper ocean waters. Two sailors, Jason and Nikki Wynn, noted this effect as they sailed from the coast of China into deep waters.

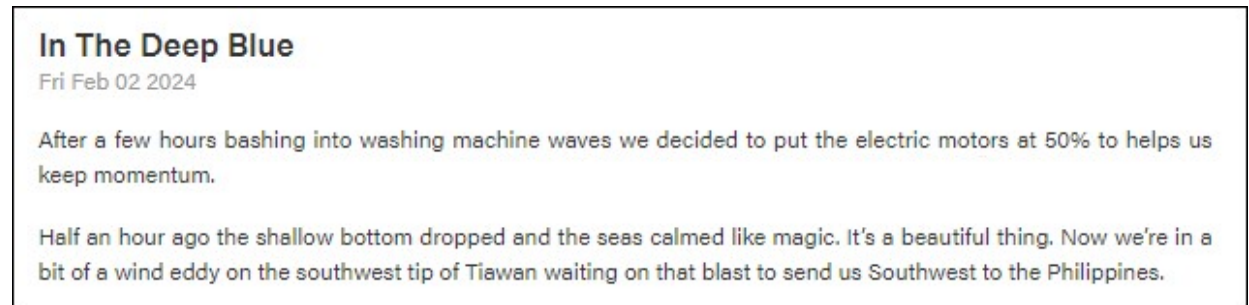


Figure 10. Reflections Can Create Interference with Surface Waves in Shallow Seas

Their sailing log observed that the coastal waters were choppy “washing machine waves” until the shallow bottom dropped and the seas “calmed like magic.” The transition from the shallow coast to deeper water can be clearly seen from the satellite view of their sailing path.

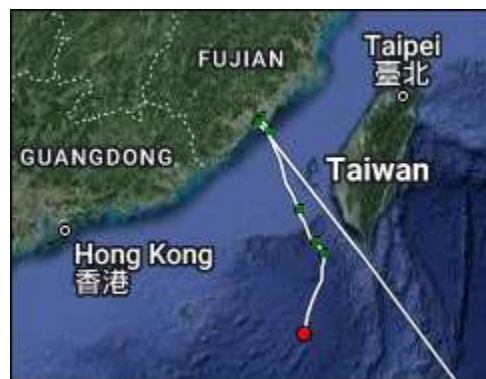


Figure 11. Satellite View Depicting Transition from Shallow Seas to Deeper Ocean

The wave tank was 30 cm high and was filled with water to the 15 cm level (i.e., halfway). The wavelengths recorded ranged from 54 cm at 90 beats per minute to 17 cm at 180 beats per minute. Even the shortest wavelengths recorded were greater than the depth of the water. Consequently, it could be expected that reflections from the bottom of the tank would interfere with surface waves, in a phenomenon similar to that which occurred when the Wynns sailed off the coast of China.

The wave lengths recorded decreased as the beats-per-minute increased. When the wave paddle is moved back-and-forth at a faster rate, it sends more waves the length of the tank in a shorter amount of time. Consequently, it would be expected that the distance between each successive wave generated by the wave paddle would decrease as the rate of movement increased.

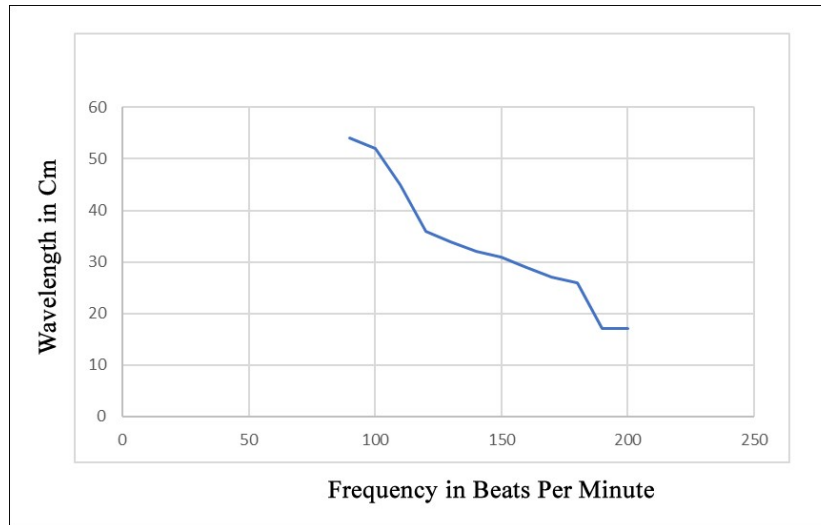
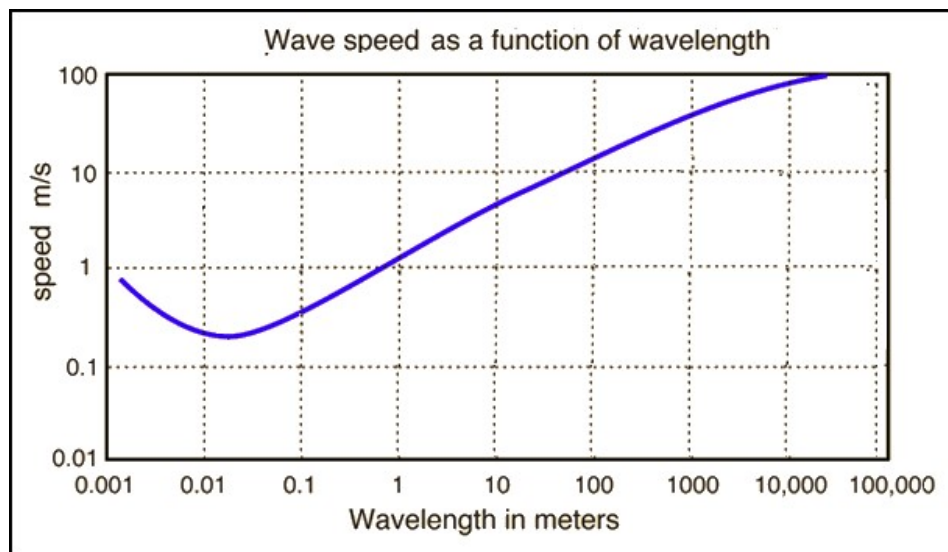


Figure 12. Wavelength Increases as Frequency in Beats per Minute Decreases

The wavelength recorded for 120 beats per minute was 36 centimeters. Since 120 beats per minute is two beats per second, the wave traveled 36 centimeters in one-half second. Consequently, the wave would travel 72 centimeters in a full second (i.e., twice as far as it would travel in half a second). In this case, the propagation speed of the wave was 72 centimeters per second or 0.7 meters per second. The speed of ocean waves generally increases as

There is a general relationship for waves that relates the speed of the wave movement to frequency and wavelength. This relationship applies to sound and light waves, other electromagnetic waves, and waves in media such as earth and water.



The phenomena described occur in waves in other mediums, such as sound waves in a tube or musical instrument. Sound waves can interfere with one another or reinforce one another just as waves in water do. The amplitude, measured as sound pressure level (perceived as loudness), is at its maximum at the resonant frequency of the tube (where maximal reinforcement of the waves occurs). The wavelength of

soundwaves decreases as frequency increases (Figure 12). A graph of wavelength in centimeters versus frequency in beats per minute approximates a straight line.

Conclusion

The measurement of the amplitude of waves in a wave tank at different frequencies provides a useful entry point for thinking about concepts that are applicable in many different contexts, because in contrast to sound waves, waves in a tank of water can be seen. Therefore, development of a frequency response curve for measurement of waves in a wave tank lays the foundation for application of similar concepts in other contexts.

References

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