Tuning a Make to Learn Monochord

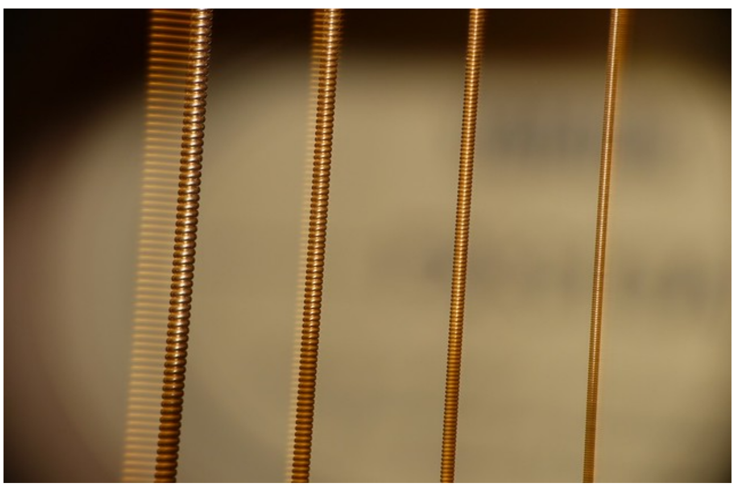
After a *Make to Learn Monochord* has been assembled, the monochord must be tuned so that the monochord string vibrates at a specific frequency. The location of notes on the monochord must also be identified in order to play a song.

Pythagoras studied the monochord to explore the nature of sound and music. The *Make to Learn Monochord* serves the same purpose more than 2,500 years later. The Nobel laureate Frank Wilczek comments that the musical rules that Pythagoras discovered "deserve … to be considered the first quantitative laws of nature ever discovered." (Wilczek, *A Beautiful Question: Finding Nature's Deep Design*)

The Nature of Sound

Vibration and Frequency

In the case of a stringed instrument, sound is produced by a back-and-forth movement of the string. The movement of a guitar string is shown in slow motion in the video below.



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The term *frequency* is used to describe the rate at which a string moves back and forth. For example, when a string is tuned to Middle C, it moves back and forth at a rate of 256 times per second.

Factors affecting Rate of Vibration

On a stringed instrument such as a piano or a guitar, frequency is affected by the length, thickness (density), and tension of the string.

Factors Affecting Frequency		
	Change	**Frequency**
Length	Increase	Decrease
Thickness	Increase	Decrease
Tension	Increase	Increase

Pitch is the perceptual correlate of frequency. Frequency is a physical phenomenon, measured in *cycles per second*.

Pitch is a perceptual phenomenon measured in *mels*. Increasing the tension on a string causes it to vibrate at a faster rate, causing the listener to perceive the note played to be higher in pitch.

Tuning the Monochord

Tuning a Monochord String

A tuning mechanism is used to increase the tension on the string by winding it more tightly. For example, the A string on a guitar tuned in the standard way would vibrate at a frequency of 110 times per second.

The tuning mechanism is used to tighten or loosen the tension on the string until it vibrates at that rate. The string is then said to be *in tune*.

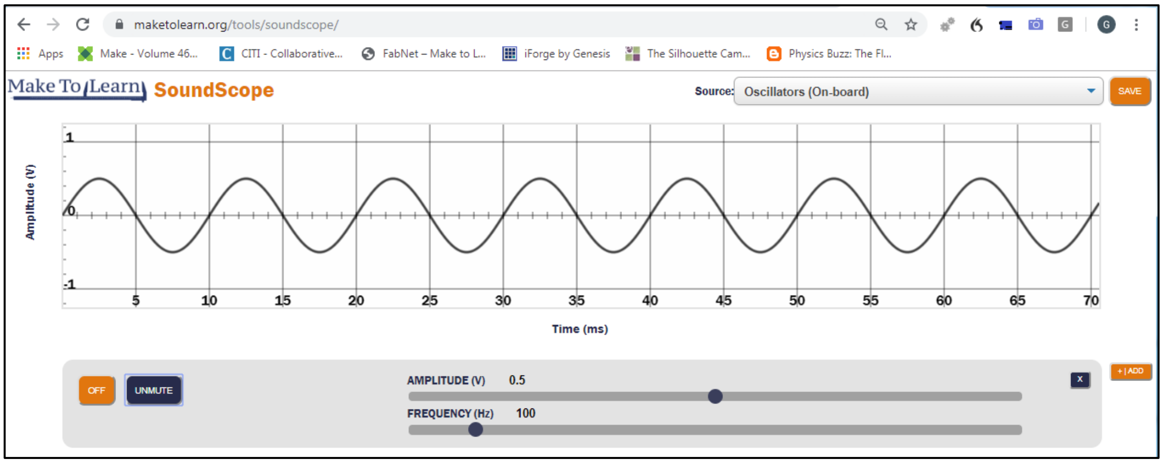
The initial frequency can be set through auditory comparison with a reference tone. The reference tone can be provided by an audio tool called *SoundScope*.

SoundScope

SoundScope is a sound synthesis and digital display tool developed in the *Make to Learn Laboratory* at the University of Virginia. It can be accessed through the *Make to Learn* web site:

www.maketolearn.org/tools/soundscope

SoundScope was designed for exploration of sound and music in conjunction with *Make to Learn Invention Kits*.

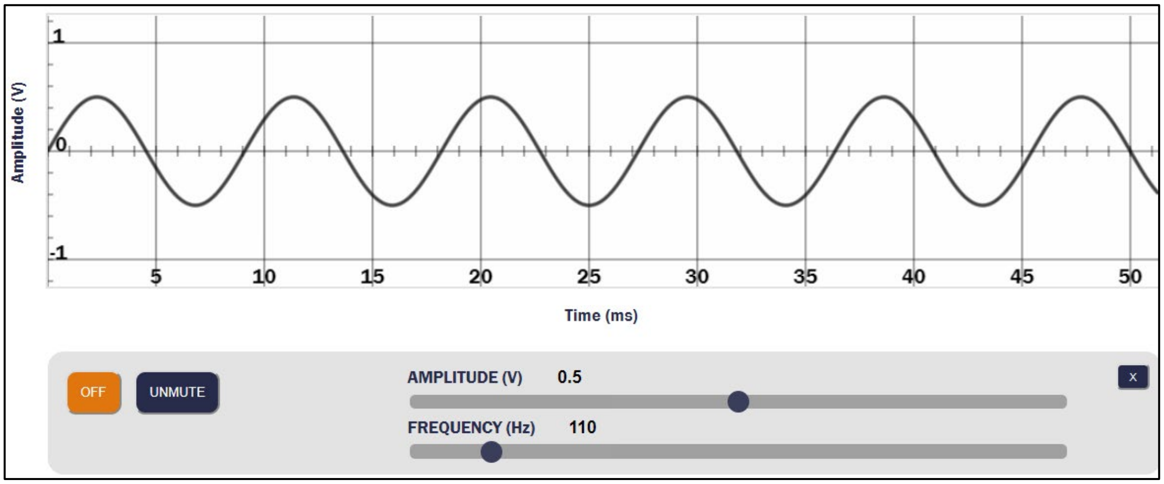


An on-board oscillator can be used to generate a tone of any frequency. This can be used as a reference tone for tuning the guitar. In the example shown, the oscillator has been set to a frequency of 100 Hz. As the display illustrates, one complete cycle of a 100 Hz tone is completed in 10 milliseconds. Since there are 1000 milliseconds in a second, this means that 100 cycles of the tone will be completed in one second.

Adjusting the Monochord Frequency

The A string on a guitar tuned in the standard way would vibrate at a frequency of 110 times per second.

To tune the monochord string to that same frequency, use *SoundScope* to generate a reference tone that is the same frequency.



Use the tuning mechanism to tighten or loosen the string of the monochord until the pitch sounds the same as the reference tone. The accuracy of the tuning can then be verified by measuring the frequency of the vibrating monochord string with *SoundScope*.

Connecting the Monochord to SoundScope

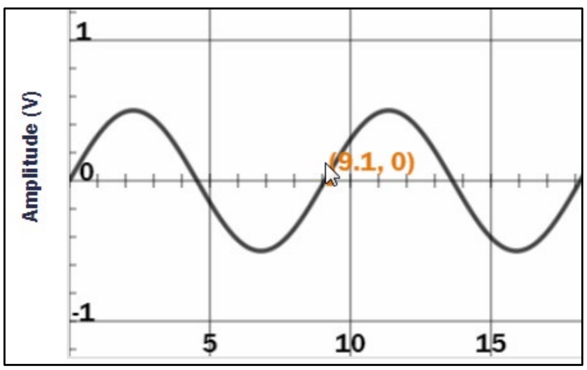
Sound generated by the *Make to Learn Electric Monochord* can be displayed and analyzed on *SoundScope*. The monochord is connected to the computer via an interface such as the U-Phoria audio interface. An audio cable is used to connect the output jack of the monochord to the input jack of the U-Phoria module. A USB cable is then used to connect the U-Phoria module to the computer.

Once the monochord is connected to a computer, launch *SoundScope*. Select the U-Phoria option from the drop-down

Source menu in the upper right-hand corner of the *SoundScope* screen. When the monochord string is plucked, the waveform generated should appear on the *SoundScope* display.

Calculating Frequency with SoundScope

SoundScope can be used to determine when the monochord string has been tuned to the desired frequency. Once the monochord string has been tuned to approximately 110 Hz (i.e., the frequency that corresponds to the musical note A), this can be verified by examining the waveform generated by the monochord on *SoundScope*.



The time required to complete one cycle of the waveform is known as the *period* of the waveform.

When the cursor is placed on the waveform on the SoundScope display, the time (in milliseconds) and amplitude of the waveform at that point in time are displayed. This feature can be used to determine the period of the waveform.

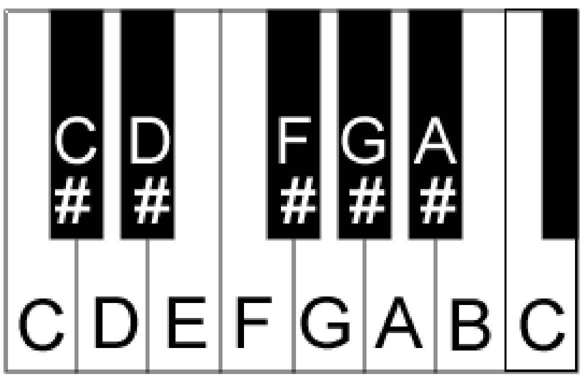
1000 milliseconds divided by 110 cycles per second equals 9.09 millisconds. The monochord will be in tune when the period of the waveform equals that amount of time. Pluck the string of the monochord and freeze the waveform on the SoundScope display to verify that its period and frequency are correct.

Identifying the Location of Notes on the Monochord

Once the monochord string has been tuned using SoundScope and the tuning mechanism, the location of notes must be identified in order to play a song.

The Traditional Western Chromatic Musical Scale

The modern Western chromatic musical scale is divided into twelve notes. This sequence of notes is known as an *octave*. On a piano, this consists of eight white notes, labeled "C, D, E, F, G, A, B" in the diagram below, and five black notes, labeled, "C#, D#, F#, G#, and A#." The second C note in the illustration is one octave higher than the first one.



Frequency and Length of Vibrating Segment

The length of a vibrating segment affects the rate of vibration of the monochord string. The rate of vibration is proportional to the length of the vibrating segment. If the monochord string is tuned to a frequency of 110 Hz, pressing down on the middle of the string will halve the length of the vibrating segment. This will cause the rate of vibration to double, increasing to 220 Hz in this instance (double the rate at which the whole string vibrates).

Locating Notes on the Monochord

To play a full octave on the monochord, beginning with A, the location of the following notes would need to be identified:

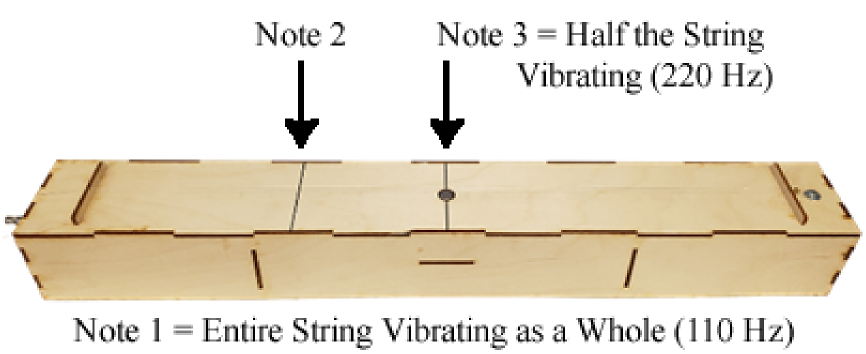
A, A#, B, C, C#, D, D#, E, F, F#, G, G#, A

This task could be simplified by breaking the task into sub tasks: (1) determine the locations for a three-note instrument, (2) determine the locations for a four-note instrument, and (3) then extend the methods used to develop solutions for these subtasks to create an algorithm for locating the position of the notes for a full octave (twelve notes).

A Three-Note Instrument

The monochord string tuned to A vibrates at 110 Hz when vibrating as whole. The string vibrating as a whole is designated as *Note 1* in the illustration below.

Dividing the string in half by pressing down in the middle of the string would cause it to vibrate one octave higher (220 Hz). This is designated as *Note 3* in the illustration below.



The task, then, is to locate the position of Note 2 on the monochord. This simplified subtask creates a new three-note scale. In a linear scale, each new note would be placed halfway between the last note places and the nut of the instrument.

However, the Traditional Western Chromatic Musical Scale is not linear, so ratios must be used to find each new note.

Locating the Position of the Second Note

In the contemporary Western chromatic scale, the ratio of the change in frequency is the same for any two successive notes. This characteristic can be used to determine the location of the Note 2.

$$\frac{\text{Note 1}}{\text{Note 2}} = \frac{\text{Note 2}}{\text{Note 3}}$$

Here, Note 1 is the length of the open string, obtained by measuring the full distance from the bridge to the nut of the instrument. Note 3 is half the distance from the bridge to the nut.

If we assume (to simplify the calculations) that the length of the entire string is 100 millimeters, then Note 3 would be 50 millimeters (half the length of Note 1).

Note 3 = 50 mm

Note 2 = ? mm

Note 1 = 100 mm

The location of Note 2 would fall somewhere between the first note and the third note. Thus, the vibrating segment for Note 2 would be less than 100 millimeters but more than 50 millimeters in this case.

$$\frac{100\text{ mm}}{\text{Note 2}} = \frac{\text{Note 2}}{50\text{ mm}}$$

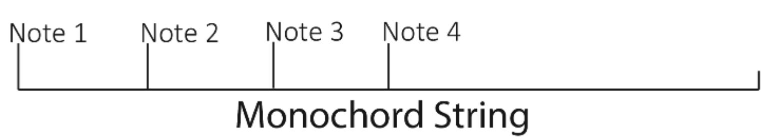
This proportion can be used to mathematically determine the solution for the placement of Note 2. In this particular instance, the distance of Note 2 from the bridge is $100/\sqrt{2}$.

For those whose last exposure to ratios and proportions was some time in the past, additional notes on the mathematical method for accomplishing this are provided in the *Appendix*.

A Four-Note Instrument

Once the solution for a three-note instrument is obtained, this method can be extended to the case of a four-note instrument. A four-note instrument would consist of (a) one note consisting of the string vibrating as a whole and (b) three additional notes. Similar to the three-note instrument, Note 1 is the length of the whole string and Note 4 is half that length. Notes 2 and 3 will fall between Note 1 and Note 4.

$$\frac{\text{Note 1}}{\text{Note 2}} = \frac{\text{Note 2}}{\text{Note 3}} = \frac{\text{Note 3}}{\text{Note 4}}$$



Since the ratios of each pair of successive notes must be the same, this information can be used to determine the location of each note, in a manner similar to the method used for the three-note instrument. Note 2 is found to be the distance of the full string divided by $\sqrt[3]{2}$, and Note 3 is Note 2 divided by $\sqrt[3]{2}$.

Developing an Algorithm

Once the solutions for a three-note instrument and a four-note instrument have been developed, this can be used as the basis for development of an algorithm that can be used to determine the locations for all twelve notes of the traditional Western musical scale. The ratio for a three-note instrument proved to be the square root of two and the ratio for a four-note instrument proved to be the cube root of two. This suggests the possibility of a pattern that could be extended for any number of notes

Appendix

Mathematical Proportions

A proportion is a statement that two ratios are equal. When two ratios are equal and one of the values is unknown (in this case, the value of the location of Note 2), this information can be used to develop a solution and determine the unknown value.

Case of the Three-Note Instrument

In the contemporary Western chromatic scale, the ratio of the change in frequency is the same for any two successive notes. In the case of a three-note instrument, the ratio of Note 1 to Note 2 and the ratio of Note 2 to Note 3 is the same.

$$\frac{\text{Note 1}}{\text{Note 2}} = \frac{\text{Note 2}}{\text{Note 3}}$$

Note 1 is the full length of the monochord string vibrating as a whole. To simplify the mathematics, we are assuming that the whole string is 100 millimeters. To span a full octave, Note 3 must be 50 millimeters, since halving the vibrating segment will double the rate of vibration.

$$\frac{100 \text{ mm}}{\text{Note 2}} = \frac{\text{Note 2}}{50 \text{ mm}}$$

Solving the Equation

The following notation is used in the description that follows:

N1 = length of vibrating string segment for Note 1 (100 mm)

N2 = length of vibrating string segment for Note 2 (unknown)

N3 = length of vibrating string segment for Note 3 (50 mm)

R = ratio of N1 to N2 and N2 to N3

By definition, N1 / N2 = N2 / N3. The ratio of both N1 / N2 and N2 / N3 = R.

$$\frac{N1}{N2} = \frac{N2}{N3} = \mathrm{R}$$

Multiplying N1 / N2 times N2 / N3 yields N1 / N3 because the N2 terms cancel one another.

$$\frac{N1}{\cancel{N2}} * \frac{\cancel{N2}}{N3} = \frac{N1}{N3}$$

Since both N1 / N2 and N2 / N3 = R, substitution of R yields the following result.

$$\mathrm{R} * \mathrm{R} = \frac{N1}{N3}$$

Since N1 = 100 mm and N3 = 50 mm, this yields the result R squared = 100 mm / 50 mm.

$$R^2 = \frac{100\ mm}{50\ mm}$$

Since 100 mm / 50 mm = 2, this results in the following:

$$\mathrm{R}^2 = 2$$

Consequently, R = the square root of 2, which is 1.4.

$$\mathrm{R}^2 = \sqrt[2]{2} = 1.414$$

Therefore, N2 = 100 mm / 1.4 which is equal to 70.7 millimeters.

$$\text{N2} = \frac{100\ mm}{1.4} = 70.721\ mm$$

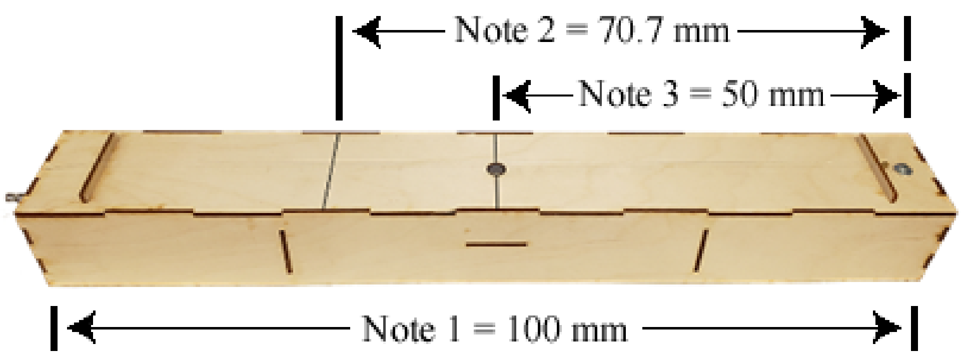
The obtained result can be verified by dividing Note 1 by Note 2 and Note 2 by Note 3. The result in each case is the same: 1.4

$$\frac{100\ mm}{70.721\ mm} = 1.414 \qquad \frac{70.721\ mm}{50\ mm} = 1.414$$

Thus, the ratio of the location of Note 1 to Note 2 is the same as the ratio of the location of Note 2 to Note 3 in each case. Since the frequency of each note is proportional to the length of the vibrating segment when the monochord string is plucked, the ratio of the frequencies of the notes is the same as well.

Marking the Locations of the Notes on the Monochord

Once the locations of the notes have been determined, they can bamarked on the monochord in the manner shown below.

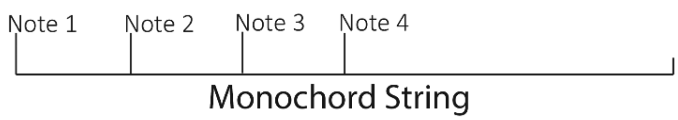


The result of 1.4 for the ratio of Note 1 to Note 2 and Note 2 to Note 3 has one other mathematical property: it is the square root of 2. This will prove to be more than a coincidence, and provides a clue for determination of the location of the notes for a four-note instrument.

Case of the Four-Note Instrument

The case of the four-note instrument is similar to the case of the three-note instrument, but in this expanded case there are three ratios rather than two.

$$\frac{\text{Note 1}}{\text{Note 2}} = \frac{\text{Note 2}}{\text{Note 3}} = \frac{\text{Note 3}}{\text{Note 4}}$$



Just as in the case of the three-note instrument, Note 1 consists of the string vibrating as a whole, and the last note (the length of the vibrating segment associated with Note 4 in this case) consists of half the string vibrating as a segment. Therefore the range from Note 1 to Note 4 once again spans a full octave.

Solving the Equation

The following notation is used in the description that follows:

N1 = length of vibrating string segment for Note 1 (100 mm)

N2 = length of vibrating string segment for Note 2 (unknown)

N3 = length of vibrating string segment for Note 3 (unknown)

N4 = length of vibrating string segment for Note 4 (50 mm)

R = ratio of N1 to N2, N2 to N3, and N3 to N4

By definition, N1 / N2 = N2 / N3 = N3 / N4.

$$\frac{N1}{N2} = \frac{N2}{N3} = \frac{N3}{N4} = R$$

Multiplying N1 / N2 times N2 / N3 times N3 / N4 yields N1 / N4.

$$\frac{N1}{\cancel{N2}} * \frac{\cancel{N2}}{\cancel{N3}} * \frac{\cancel{N3}}{N4} = \frac{N1}{N4}$$

Since N1 / N2, N2 / N3, and N3 / N4 all equal R, substitution of R yields the following result.

$$\mathrm{R} = \mathrm{R} = \mathrm{R} = \frac{N1}{N4}$$

Since N1 = 100 millimeters and N4 = 50 millimeters, R cubed equals 100 mm / 50 mm.

$$\mathrm{R}^3 = \frac{100\ mm}{50\ mm}$$

Thus R cubed is equal to 2.

$$R^3 = 2$$

Consequently, the ratio R is equal to the cube root of 2;.

$$\mathrm{R}^3 = \sqrt[3]{2} = 1.26$$

The length of the monochord string associated with Note 1 is 100 millimeters and the cube root of 2 is 1.26. Substituting these values into the equation, yields the following:

$$\frac{100\ mm}{1.26} = Note\ 2$$

Dividing 100 millimeters by 1.26 yields the solution. The length of the vibrating segment associated with Note 2 is 79.3 millimeters.

$$70.3\ mm = Note\ 2$$

Since the ratios of the change in frequency between Note 1 divided by Note 2 and Note 2 divided by Note 3 are equal, we can conclude that Note 3 is equal to Note 2 divided by the cube root of 2.

$$\frac{Note\ 2}{\sqrt[3]{2}} = Note\ 3$$

Consequently Note 3 is equal to 79.3 millimeters divided by 1.26.

$$\frac{79.3\ mm}{1.26} = Note\ 3$$

We can therefore conclude that the length of the vibrating segment associated with Note 3 is 62.9 millimeters.

$$62.9\ \text{mm} = Note\ 3$$

We previously determined that the length of the vibrating segment associated with Note 4 was 50 millimeters. If the pattern obtained is correct, dividing 62.9 by the cube root of 2 should also yield 50 millimeters:

$$\frac{62.9\ mm}{\sqrt[3]{2}} = 50\ mm = Note\ 4$$

Returning to our original ratios, we find that dividing each note by the note that follows it yields the same ratio: the cube root of 2 (i.e., 1.26).

$$\frac{100\ mm}{79.3\ mm} = \frac{79.3\ mm}{62.9\ mm} = \frac{62.9\ mm}{50.0\ mm} = 1.26$$

This result will allow us to mark the correct location of each note on the monochord.

$$\frac{\text{Note 1}}{\text{Note 2}} = \frac{\text{Note 2}}{\text{Note 3}} = \frac{\text{Note 3}}{\text{Note 4}}$$